J. K. SHAH CLASSES

FYJC TEST – MATHEMATICS & STATISTICS

SOLUTION SET

Q1. Attempt any FOUR of the following **(3 MARK EACH)** **(12 marks)**

01. there are 15 players including A , B & C . Find the number of ways in which cricket team of 11 can be chosen if A is selected as captain & at the same time B is not available

SOLUTION

Since A is selected as captain & at the same time B is not available , the remaining 10 players have to be selected from the remaining 13 players .

This can be done in $= {}^{13}C_{10} = {}^{13}C_3 = 286$ ways

02. Find the number of straight lines obtained by joining 10 points on a plane , if four points are collinear

SOLUTION

10 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{10}C_2 = 45$

But 4 points are collinear

Number of lines wrongly counted in these 4 collinear points $= {}^{4}C_2 = 6$ instead of 1

Hence

actual lines that can be drawn $= 45 - 6 + 1 = 40$

03. the probability that a student will get a gold medal is 0.4 and that he will not get a silver medal is 0.7 . If the probability of getting at least one medal is 0.6 , what is the probability that he will get neither of the medals

SOLUTION :

A : student will get 'GOLD' medal $P(A) = 0.4$

B : student will get 'SILVER' medal $P(B) = 1 - 0.7 = 0.3$

$A \cup B$: student will get at least one medal $P(A \cup B) = 0.6$

E $\equiv$ student will get neither of the medals

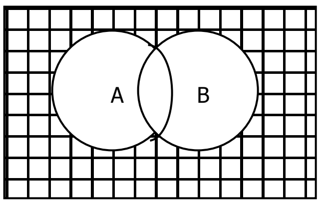
$E \equiv A' \cap B'$

$P(E) = P(A' \cap B')$

$= 1 - P(A \cup B)$

$= 1 - 0.6$

$= 0.4$



04. a problem is given to three students A ,B , C whose chances of solving it are 1/2 , 1/3 & 1/4 respectively . Find the probability that the problem will be solved

SOLUTION :

A : student A can solve a problem P(A) = $^1/_2$, P(A') = $^1/_2$

B : student B can solve a problem P(B) = $^1/_3$, P(B') = $^2/_3$

B : student B can solve a problem P(B) = $^1/_4$, P(C') = $^3/_4$

E ≡ problem is solved

E' ≡ problem is not solved

E' ≡ $A' \cap B' \cap C'$

$P(E') = P(A' \cap B' \cap C')$

$= P(A') \times P(B') \times P(C')$

$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{4}$

$P(E) = 1 - P(E')$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

05. An urn contains 12 items of which 4 are defective . Two items are drawn at random from the urn one after the other without replacement . Find the probability that both items are non defective

SOLUTION :

12 items - 4 defective & 8 non defective

exp : Two items are drawn at random from the urn one after the other without replacement

E ≡ both items are non defective .

E ≡ First item is non - defective AND second item is non - defective

E ≡ $A \cap B$

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B \mid A)$

$= \frac{8}{12} \times \frac{7}{11}$

$= \frac{14}{33}$

Q2. **Attempt any TWO of the following** **(4 MARK EACH)** **(8 marks)**

01. a question paper consists of 11 questions divided into two sections I and II . Section I consists of 5 questions and section II consists of 6 questions . In how many ways can a student select 6 questions taking at least 2 questions from each section

SOLUTION

Section I consists of 5 questions and section II consists of 6 questions

student select 6 questions taking at least 2 questions from each section

Case 1 : students selects 2 Q's from Section I & 4 Q's from Section II

This can done in $= {}^5C_2 \times {}^6C_4 . \quad = {}^5C_2 \times {}^6C_2 .$

$= 10 \times 15 = 150$ ways

Case 2 : students selects 3 Q's from Section I & 3 Q's from Section II

This can done in $= {}^5C_3 \times {}^6C_3 . \quad = {}^5C_2 \times {}^6C_3 .$

$= 10 \times 20 = 200$ ways

Case 3 : students selects 4 Q's from Section I & 2 Q's from Section II

This can done in $= {}^5C_5 \times {}^6C_2 . \quad = {}^5C_1 \times {}^6C_2 .$

$= 5 \times 15 = 75$ ways

By fundamental principle of ADDITION

Total ways $= 425$

02. The probability that A can shoot a target is 3 / 4 and the probability that B can shoot is 3 / 5 . If A and B shoot independently of each other , find the probability that exactly one of A and B shoot the target

SOLUTION

A : A will shoot the target $P(A) = {}^{3}/_{4}$, $P(A') = {}^{1}/_{4}$

B : B will shoot the target $P(B) = {}^{3}/_{5}$, $P(B') = {}^{2}/_{5}$

$E \equiv$ exactly one of A and B shoot the target

$E \equiv E_1 \cup E_2$ where

$E_1 \equiv$ A shoots and B fails

$E_1 \equiv A \cap B'$

$P(E_1) = P(A \cap B')$

$= P(A) \times P(B')$

$= {}^{3}/_{4} \times {}^{2}/_{5} = {}^{6}/_{20}$

$E_2 \equiv$ A fails and B shoots

$E_2 \equiv A' \cap B$

$P(E_1) = P(A' \cap B)$

$= P(A') \times P(B)$

$= {}^{1}/_{4} \times {}^{3}/_{5} = {}^{3}/_{20}$

Now

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ ....E1 & E2 are mutually exlusive

$= {}^{6}/_{20} + {}^{3}/_{20}$

$= {}^{9}/_{20}$

03. a bag contains 5 white balls & 4 Black balls . A second bag contains 4 white balls & 6 black . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find the probability that it is black ball

SOLUTION

exp : One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag

E ≡ ball drawn is BLACK

$E \equiv E_1 \cup E_2$

Where

E_1 ≡ White ball is transferred from Bag 1 to Bag 2 AND a Black ball is drawn from bag 2

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B \mid A)$

$= \frac{5}{9} \times \frac{6}{11}$

NEW CONFIGURATION OF BAG 2

5 white, 6 black

$= \frac{30}{99}$

E_2 ≡ Black ball is transferred from Bag 1 to Bag 2 AND a Black ball is drawn from bag 2

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B \mid A)$

$= \frac{4}{9} \times \frac{7}{11}$

NEW CONFIGURATION OF BAG 2

4 white, 7 black

$= \frac{28}{99}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXHAUSTIVE

$= \frac{30}{99} + \frac{28}{99}$

$= 58/99$

Q3. Attempt any FOUR of the following (3 MARK EACH) (12 marks)

01. Prove $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

SOLUTION

$$\text{LHS} = \sqrt{\frac{\cos^2 \theta/2 + \sin^2 \theta/2 + 2\sin \theta/2 . \cos \theta/2}{\cos^2 \theta/2 + \sin^2 \theta/2 - 2\sin \theta/2 . \cos \theta/2}}$$

$$= \sqrt{\frac{(\cos \theta/2 + \sin \theta/2)^2}{(\cos \theta/2 - \sin \theta/2)^2}}$$

$$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2}$$

Dividing Numerator & Denominator by $\cos \theta/2$

$$= \frac{\dfrac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2}}{\dfrac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2}}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2} = \text{RHS}$$

$$\text{RHS} = \tan (\pi/4 + \theta/2)$$

$$= \frac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2}$$

LHS = RHS

02. Prove $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

SOLUTION

LHS

$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2\,.\,2\cos^2 4\theta}}}$

$= \sqrt{2+\sqrt{2+\sqrt{4\cos^2 4\theta}}}$

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$= \sqrt{2+\sqrt{2\,.2\cos^2 2\theta}}$

$= \sqrt{2+\sqrt{4\cos^2 2\theta}}$

$= \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2\,.2\cos^2\theta}$

$= \sqrt{4\cos^2\theta}$

$= 2\cos\theta$

03. Prove $\dfrac{\cos 3A - 2.\cos 5A + \cos 7A}{\cos A - 2.\cos 3A + \cos 5A} = \cos 2A - \sin 2A \cdot \tan 3A$

SOLUTION

$$\text{LHS} = \frac{\cos 7A + \cos 3A - 2.\cos 5A}{\cos 5A + \cos A - 2.\cos 3A}$$

$$= \frac{2\cos\left(\frac{7A+3A}{2}\right).\cos\left(\frac{7A-3A}{2}\right) - 2.\cos 5A}{2\cos\left(\frac{5A+A}{2}\right).\cos\left(\frac{5A-A}{2}\right) - 2.\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cos 2A - 2.\cos 5A}{2\cos 3A \cdot \cos 2A - 2.\cos 3A}$$

$$= \frac{2\cos 5A \cdot (\cos 2A - 1)}{2\cos 3A \cdot (\cos 2A - 1)}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \frac{\cos (3A + 2A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos 2A - \sin 3A \cdot \sin 2A}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos 2A}{\cos 3A} - \frac{\sin 3A \cdot \sin 2A}{\cos 3A}$$

$$= \cos 2A - \sin 2A \cdot \tan 3A$$

$$= \text{RHS}$$

04. Prove $\sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ = 3/16$

SOLUTION

LHS $= \sin 20^\circ . \sin 40^\circ . \sin 60^\circ . \sin 80^\circ$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \sin (60 - 20)^\circ . \sin (60 + 20)^\circ$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . (\sin 60 \cos 20 - \cos 60 \sin 20) . (\sin 60 \cos 20 + \cos 60 \sin 20)$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \left(\frac{\sqrt{3}\cos 20}{2} + \frac{1}{2}\sin 20\right) . \left(\frac{\sqrt{3}\cos 20}{2} + \frac{1}{2}\sin 20\right)$$

$$= \frac{\sqrt{3}}{2} . \sin 20^\circ . \left(\frac{3}{4}\cos^2 20 - \frac{1}{4}\sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3\cos^2 20 - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3(1 - \sin^2 20) - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . \left(3 - 3\sin^2 20 - \sin^2 20\right)$$

$$= \frac{\sqrt{3}}{8} . \sin 20^\circ . (3 - 4\sin^2 20)$$

$$= \frac{\sqrt{3}}{8} . (3\sin 20^\circ - 4\sin^3 20)$$

$$= \frac{\sqrt{3}}{8} . \sin (3 \times 20^\circ)$$

$$= \frac{\sqrt{3}}{8} . \sin 30^\circ$$

$$= \frac{\sqrt{3}}{8} . \frac{1}{2}$$

$$= \frac{3}{16} \qquad = \text{RHS}$$

05. Prove $\cos 3^\circ \cdot \cos 6^\circ \cdot \cos 12^\circ \cdot \cos 24^\circ = \dfrac{\cos 42^\circ}{16 \sin 3^\circ}$

SOLUTION

We Prove

$16 \sin 3^\circ \cdot \cos 3^\circ \cdot \cos 6^\circ \cdot \cos 12^\circ \cdot \cos 24^\circ = \cos 42^\circ$

LHS

$= 16 \sin 3^\circ . \cos 3^\circ . \cos 6^\circ . \cos 12^\circ . \cos 24^\circ$

$= 2.2.2.\underline{2\sin 3^\circ . \cos 3^\circ} \cos 6^\circ . \cos 12^\circ . \cos 24^\circ$

$= 2.2.\underline{2.\sin 6^\circ \cos 6^\circ} \cos 12^\circ . \cos 24^\circ$

$= 2.\underline{2 \sin 12^\circ \cos 12^\circ} \cos 24^\circ$

$= 2 \sin 24^\circ \cos 24^\circ$

$= \sin 48^\circ$

$= \sin(90^\circ - 42^\circ)$

$= \cos 42^\circ$

$=$ RHS

Q4. Attempt any TWO of the following (4 MARK EACH) (8 marks)

01. Prove $\dfrac{\sin 3\theta.\cos 5\theta - \sin \theta.\cos 7\theta}{\sin \theta.\sin 7\theta - \cos 3\theta.\cos 5\theta} = \tan 2\theta$

SOLUTION

$$\text{LHS} = \frac{2.\cos 5\theta.\sin 3\theta - 2.\cos 7\theta.\sin \theta}{2.\sin 7\theta.\sin \theta + 2.\cos 5\theta.\cos 3\theta}$$

$$= \frac{\sin(5\theta + 3\theta) - \sin(5\theta - 3\theta) - [\sin(7\theta + \theta) - \sin(7\theta - \theta)]}{\cos(7\theta - \theta) - \cos(7\theta + \theta) + \cos(5\theta + 3\theta) + \cos(5\theta - 3\theta)}$$

$$= \frac{\sin 8\theta - \sin 2\theta - [\sin 8\theta - \sin 6\theta]}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 8\theta - \sin 2\theta - \sin 8\theta + \sin 6\theta}{\cos 6\theta - \cos 8\theta + \cos 8\theta + \cos 2\theta}$$

$$= \frac{\sin 6\theta - \sin 2\theta}{\cos 6\theta + \cos 2\theta}$$

$$= \frac{2\cos\left[\dfrac{6\theta + 2\theta}{2}\right].\sin\left[\dfrac{6\theta - 2\theta}{2}\right]}{2\cos\left[\dfrac{6\theta + 2\theta}{2}\right].\cos\left[\dfrac{6\theta - 2\theta}{2}\right]}$$

$$= \frac{2.\cos 4\theta.\sin 2\theta}{2.\cos 4\theta.\cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \cot 2\theta$$

02. Prove : $\cos 20^\circ . \cos 100^\circ + \cos 100^\circ . \cos 140^\circ - \cos 140^\circ . \cos 200^\circ = -3/4$

SOLUTION

WE PROVE :

$2.\cos 20^\circ . \cos 100^\circ + 2.\cos 100^\circ . \cos 140^\circ - 2.\cos 140^\circ . \cos 200^\circ = -3/2$

LHS $= 2.\cos 100^\circ . \cos 20^\circ + 2.\cos 140^\circ . \cos 100^\circ - 2.\cos 200^\circ . \cos 140^\circ$

$= \cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \left(\cos 340^\circ + \cos 60^\circ\right)$

$= \cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ$

Now : $\cos 120 = \cos(180 - 60) = -\cos 60 = -1/2$
(II QUAD)

$\cos 240 = \cos(180 + 60) = -\cos 60 = -1/2$
(III QUAD)

$\cos 340 = \cos(360 - 20) = +\cos 20$
(IV QUAD)

BACK IN SUM

$= -1/2 + \cos 80^\circ - 1/2 + \cos 40^\circ - \cos 20^\circ - 1/2$

$= -3/2 + \cos 80^\circ + \cos 40^\circ - \cos 20^\circ$

$= -3/2 + 2\cos\left[\frac{80+40}{2}\right]^\circ . \cos\left[\frac{80-40}{2}\right]^\circ - \cos 20^\circ$

$= -3/2 + 2.\cos 60^\circ \cos 20^\circ - \cos 20^\circ$

$= -3/2 + 2.(1/2)\cos 20^\circ - \cos 20^\circ$

$= -3/2 + \cos 20^\circ - \cos 20^\circ$

$= -3/2$

03. Prove : $\dfrac{\sec 8A - 1}{\sec 4A - 1} = \dfrac{\tan 8A}{\tan 2A}$

SOLUTION

$$\text{LHS} = \frac{\sec 8A - 1}{\sec 4A - 1}$$

$$= \frac{\dfrac{1}{\cos 8A} - 1}{\dfrac{1}{\cos 4A} - 1}$$

$$= \frac{\dfrac{1 - \cos 8A}{\cos 8A}}{\dfrac{1 - \cos 4A}{\cos 4A}}$$

$$= \frac{1 - \cos 8A}{\cos 8A} \times \frac{\cos 4A}{1 - \cos 4A}$$

$$= \frac{2 \sin^2 4A}{\cos 8A} \times \frac{\cos 4A}{2\sin^2 2A}$$

$$= \frac{2. \sin 4A . \cos 4A}{\cos 8A} . \frac{\sin 4A}{2\sin^2 2A}$$

$$= \frac{\sin 8A}{\cos 8A} . \frac{2.\sin 2A . \cos 2A}{2 \sin^2 2A}$$

$$= \frac{\sin 8A}{\cos 8A} . \frac{\cos 2A}{\sin 2A}$$

$$= \tan 8A . \cot 2A$$

$$= \frac{\tan 8A}{\tan 2A}$$